weak CY (i.e. weak wrt. Y. V(corresponds))

compact case

non. smooth case

$A \in \mathrm{Perf}(\mathbb{C}\text{-mod})$

$A \in \mathrm{Perf}(A \otimes A^{op}\text{-mod})$

Def: CY str. solution of MC eqn.

in necklace Lie alg.

$(,) : \mathrm{Sym}^2 A \to \mathbb{C}$

$\prod_{n \geq 1} \mathrm{Hom}(A^{\otimes n}_{\mathbb{Z}/n}, \mathbb{C})$

$\{X, Y\} = \sum X \cdots$

?

Ginzburg:

$A^\vee := R\mathrm{Hom}_{A \otimes A^{op}}(A, A \otimes A^{op})$

$\exists\ A^\vee \sim A[-d]$

Classif: $\leftrightarrow$ A_∞ alg. A of hom. unital

+ class $\alpha : HC_\cdot(A) \to \mathbb{C}$

s.t. composition $HH_\cdot(A) \to HC_\cdot(A) \xrightarrow{\alpha} \mathbb{C}$

is element of $(HH_\cdot(A))^* = R\mathrm{Hom}_{A\text{-}A}(A, A^*)$ induces iso $A[d] \sim A^*$

Classif. class $\beta \in HC^-(A)$ s.t.

$\tilde{\beta} \in HH_\cdot(A) = R\mathrm{Hom}_{A\text{-}A}(A^\vee, A)$ is q.is $A^\vee \to A[-d]$

PROP: $H = H_\cdot(A, A)$ Hochschild homology

$H_\cdot(M_{g, n_i, n_i}) \otimes H^{\otimes n_i} \to H^{\otimes n_o}$ $n_i \geq 1$, $n_o \geq ?$

exc. $g = 1$, $(1, 0), \ldots$

PROP

$H_\cdot(M_{g, n_i, n_o}) \otimes H^{\otimes n_i} \to H^{\otimes n_o}$ $n_i \geq 0$, $n_o \geq 1$

Def of weak CY: contains both compact & non. smooth cases

$\boxed{\text{No finiteness assumptions!}}$ $\boxed{\text{Derived Morita invariant!}}$

weak CY str. on $\mathbb{Z}$-g. space A is

collection of $m_{n_1,\dots,n_k} : A^{\otimes n_1} \otimes \dots \otimes A^{\otimes n_k} \to A^{\otimes k}$

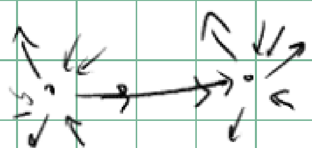
$\forall\ k \geq 1 \quad n_1 \dots n_k \geq 0$

vanishing: $m_0 = 0$ (if $k=1$ $n_1 = 0$) $(m^k_\emptyset = 0)$

$\boxed{\begin{array}{l} m_\emptyset = 0 \\ m_0 = 0 \end{array}}$

Some cyclicity and degree condition dep. on $d \in \mathbb{Z}$

Maurer-Cartan eqn. $[m_{(\cdot)}, m_{(\cdot)}] = 0$



Interpretations. $C^{(k)} = \mathrm{Hom}_{A^{\otimes k}-(A^{\otimes k})}\left(A^{\otimes k}, \sigma_* (A^{\otimes k})\right)$

diag. bimodule — gen. of $(12\cdots k)$ of $\mathbb{Z}/k$

$\mathbb{Z}/k$ action.

$C^{(1)}, C^{(2)}, \ldots$

if A smooth compact

Serre functor S $\quad S^{-1} = A^\vee \otimes_A$. $\quad S = A^* \otimes_A$

$$C^{(k)} = \mathrm{Hom}_{\mathrm{Fun}}\left(Id, S^{1-k} \right) = \mathrm{Hom}_{\mathrm{Fun}}(S^{k-1}, Id)$$

$k=2$ non deg. ⇐ smooth $\quad Id \to S^{-1}$ $\quad A \to A^!$ $\qquad A^! \to A$

inverse morphism to one in definition.

$\mathbb{Z}/k\mathbb{Z}$ action on $C^{(k)}$ — satisfies of power of K^{1-k}

for geom $D^b(\mathrm{Coh}\, X)$ $\quad \pm 1$ eigenvalue? why?

Compare: cyclic $A^{\otimes k} \otimes_{A^e} \sigma(A^{\otimes k})$

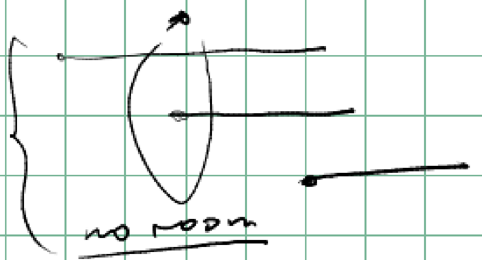
$\mathbb{Z}/k$. it is trivial!!

(Kaledin)

Examples of weak CY:

1. X smooth scheme $/\mathbb{C}$, $\dim X = d$

+ section of K_X^{-1} $\rightsquigarrow$ weak CY



Mirror dual:
(should be)

FS category of $\begin{array}{c} Y \\ \downarrow w \\ \mathbb{C} \end{array}$ Y symplectic.

finite dim. CY

= weak CY on + $\dim A < \infty$

s.t. $A^* \to A$ is q-is!

2. L: finite CW complex, connected, x_0

$A = \mathrm{Chains}(\mathrm{Loops}(L, x_0))$

hom. smooth

$H_\bullet(L) \rightsquigarrow$ weak CY of..

Poincaré duality space $\leftrightarrow$ CY hom. smooth

More generally

cycl.

L sing. [illegible] $\subset X$

weakly [illegible]

$\rightsquigarrow A$ a gen. of this

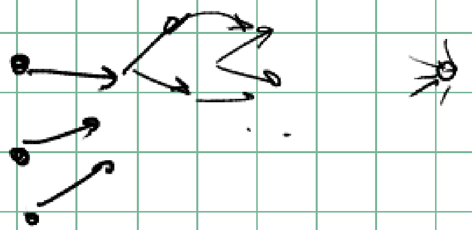
CY hom smooth

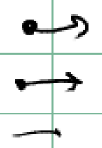
Calculus:

∀ weak CY hom. unital.

$H_\bullet(M_{g,n_{in},n_{out}}) \otimes H^{\otimes n_{in}} \to H^{\otimes n_{out}}$ $\quad g \geq 0$, $n_{in}, n_{out} \geq 1$

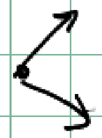
Acyclic oriented graphs

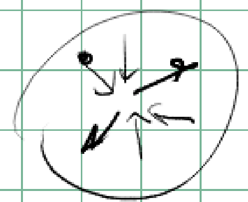




$n_{?}$ $\deg = 1$

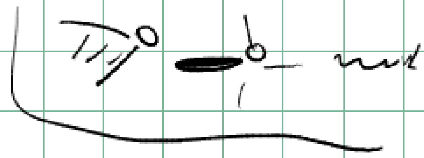
∀ inner vertex





each

$\deg_{in} \geq 1$



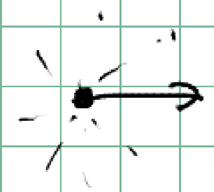
not

$\deg_{out} \geq 1$

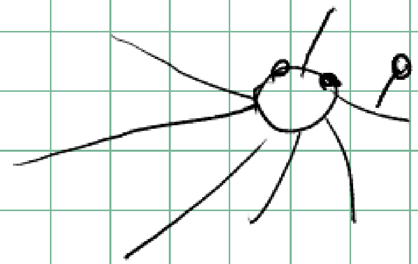
if $\deg_{out} = 1$

then $\deg_{in} \geq 2$

rough picture



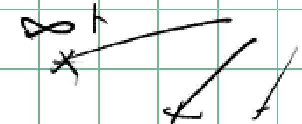
Why cell decomposition of $M_{g,n_{in},n_{out}}$?



non oriented graphs

metric

∞

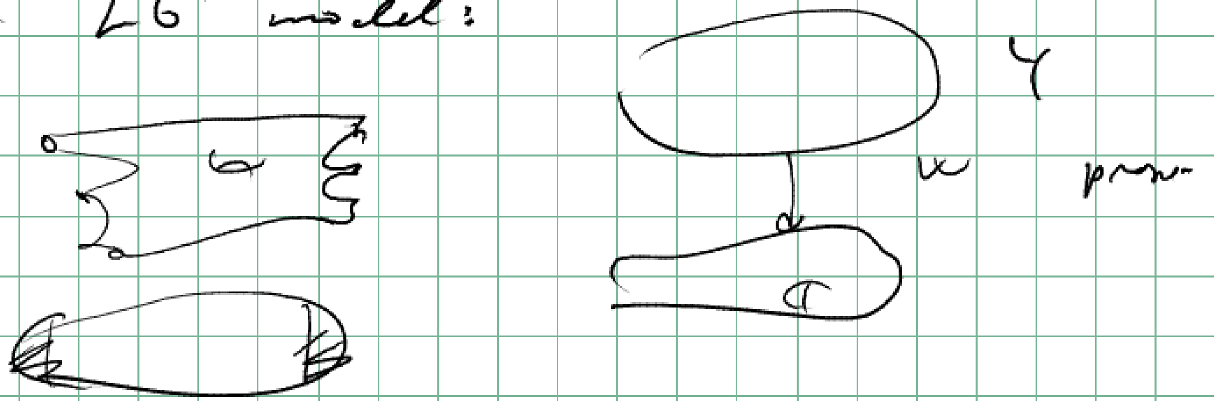


distance from "results")

gradient flow!

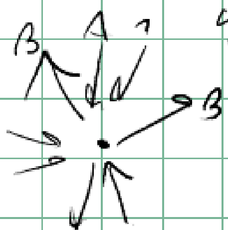
Application: 1) alg. def of "string topology"

2) For LG model:



Bonus: weak CY for a category:

Morphisms of $A \to B$



given by (f) $\mathrm{Hom}(A^{\otimes n_1} \otimes \ldots \otimes A^{\otimes n_r}, B^{\otimes r})$

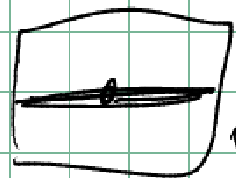
$n_i \geq 1$, $r \geq 1$

MC equation

page 6

Origin: A fin. dim.

Commutative analogue: Space of repr. of Bor.



$L \subset X \sim T^*L$
Lar. symplectic

Lag. $\subset$ Sympl
formal neighborhood

\+ ham. vector field Q tangent to L

$Q^2 = 0$ $\qquad Q|_L = 0$.

Lie_∞ algebra

Micromorphism (A. Weinstein, ...)

(q_i, p_i) $\qquad (\tilde{q}_i, \tilde{p}_i)$

Lagr. sub. $\subset \overline{T^*L_1} \oplus T^*L_2$ $\qquad L_f$

$f : L_1 \to L_2$

$F(q_i, \tilde{p}_i)$

$L_f \cap (L_1 \times L_2) =$ graph L wrt unit. 1?

$f(q, \tilde{p}) \circ g(\tilde{q}, \hat{p})$

$=$ crit. value $f + g - \tilde{p}\tilde{q}$.

Composable!

Most difficult part: classif. result for smooth CY:

Similar comm. result:

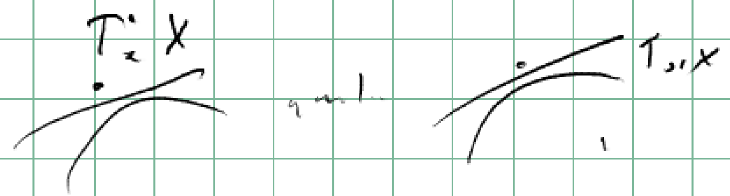
M supermfld γ_1 homol. vector field $[\gamma_1, \gamma_1] = 0$

- $(\gamma_2, \gamma_3, \ldots)$ polyvector field, $[\sum_{i \geq 1} \gamma_i, \sum_{i} \gamma_i] = 0$
- $(\omega_2, \omega_3, \ldots)$ forms

$$(d + \mathrm{Lie}_{\gamma_1})(\sum \omega_i) = 0$$

Non-degen. γ_2 inverse ω_2 ω_1 to homot. then 1:1

Legendre transform:



page 8